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TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA

TRECOM TECHNICAL REPORT 63-74

SOME CHARACTERISTICS OF THE TURBULENT BOUNDARY LAYER

Task 1D121401A14203
(Formerly Task 9R38-11-009-03)
Contract DA 44-177-AMC-892(T)

December 1963

prepared by:

MISSISSIPPI STATE UNIVERSITY
The Aerophysics Department
State College, Mississippi

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This report represents a theoretical investigation of the turbulent boundary layer. The purpose of the examination was to derive relations on the characteristics of the boundary layer and to compare them with experimental data.

This report is published for the exchange of information and the stimulation of ideas.

Gary N. Smith
GARY N. SMITH
Project Engineer

Paul J. Carpenter
PAUL J. CARPENTER
Group Leader
Applied Aeronautical Engr Group

APPROVED.

FOR THE COMMANDER:

Larry M. Hewin
LARRY M. HEWIN
Technical Director

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Aerophysics Research Note No. 18

Prepared by
The Aerophysics Department
Mississippi State University
State College, Mississippi

for
U. S. ARMY TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA

CONTENTS

ILLUSTRATIONS

SYMBOLS

δ	Boundary layer thickness, feet
θ	Momentum loss thickness, $\int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy$, feet
δ^*	Displacement thickness, $\int_0^\infty \left(1-\frac{u}{U}\right)dy$, feet
δ^{**}	Energy loss thickness, $\int_0^\infty \frac{u}{U}\left(1-\frac{u^2}{U^2}\right)dy$, feet
H	Boundary layer shape parameter $\frac{\delta^*}{\theta}$
$\bar{H}$	Boundary layer shape parameter $\frac{\delta^{**}}{\theta}$
α	Wall function
β	Wake function
U_τ	Friction velocity, $\sqrt{\frac{\tau_0}{\rho}}$, feet per second
ρ	Fluid density, slugs per cubic foot
τ_0	Surface shearing stress, pounds per square foot
U_w	Velocity component due to the wake
Π	Coles' wake parameter
f_1, f_2	Boundary layer functions
R_θ	Reynolds number based on momentum thickness
ν	Kinematic viscosity, square feet per second

INTRODUCTION

From an examination of the turbulent boundary layer profile, several relations are derived relating various parameters of a turbulent boundary layer.

$$\frac{\theta}{\delta} = \frac{\delta^*}{\delta} - f_1 \frac{\delta^{*2}}{\delta^2}$$

$$\frac{\delta^{**}}{\delta} = 2\frac{\delta^*}{\delta} - 3f_1 \frac{\delta^{*2}}{\delta^2} + f_2 \frac{\delta^{*3}}{\delta^3}$$

$$H = \frac{1}{1 - f_1 \frac{\delta^*}{\delta}}$$

$$\bar{H} = \frac{2\frac{\delta^*}{\delta} - 3f_1 \frac{\delta^{*2}}{\delta^2} + f_2 \frac{\delta^{*3}}{\delta^3}}{\frac{\delta^*}{\delta} - f_1 \frac{\delta^{*2}}{\delta^2}}$$

These relations are shown to have limits of application, and these limits are defined. Finally, the theories are compared with experimental data and show good agreement.

DISCUSSION

A description of the turbulent boundary layer profile has recently been developed which utilizes the law of the wall (Reference 1), Coles' wake function (Reference 2), and a third function in regions away from the wall (Reference 3).

The profile description may be written as

$$\frac{u}{U} = 1 - \frac{U_\tau}{U}\left(\alpha + \beta \frac{U_w}{U_\tau}\right) \tag{1}$$

where α and β (Figure 1) are functions only of the nondimensional height, y/δ, in the boundary layer and the term U_w/U_τ is a parameter describing the wake velocity. The term U_w/U_τ is related to Coles' π in the following manner:

$$\frac{U_w}{U_\tau} = \frac{2}{k}\pi$$

or, taking k, Prandtl's mixing length constant, as 0.412,

$$\frac{U_w}{U_\tau} = 4.85\pi .$$

As in the case of Coles' π, if $\frac{U_w}{U_\tau}$ is a constant throughout a boundary layer, the boundary layer is an "equilibrium" boundary layer as defined by Clauser (Reference 4). Figure 1 illustrates the present nomenclature.

Equation 1 may be used to determine the displacement, momentum, and energy thicknesses by proper integration across the boundary layer. When this is done, it is found that

$$\frac{\delta^* U}{\delta U_\tau} = \left(2.3 + 0.5 \frac{U_w}{U_\tau}\right) \tag{2}$$

$$\frac{\theta U}{\delta U_\tau} = \left(2.3 + 0.5 \frac{U_w}{U_\tau}\right) - \frac{U_\tau}{U}\left(1.12 + 3.78 \frac{U_w}{U_\tau} + 0.38 \frac{U_w^2}{U_\tau^2}\right) \tag{3}$$

$$\frac{\delta^{**} U}{\delta U_\tau} = \left(4.604 + \frac{U_w}{U_\tau}\right) - \frac{U_\tau}{U}\left(33.36 + 11.35 \frac{U_w}{U_\tau} + 1.138 \frac{U_w^2}{U_\tau^2}\right) + \tag{4}$$

$$\frac{U_\tau^2}{U^2}\left(71.4 + 18.8 \frac{U_w}{U_\tau} + 4.8 \frac{U_w^2}{U_\tau^2} + 0.303 \frac{U_w^3}{U_\tau^3}\right).$$

If equation 2 is used to make a substitution for the first term of the right-hand side of the equation, then

$$\frac{\theta U}{\delta U_\tau} = \frac{\delta^* U}{\delta U_\tau} - \frac{U_\tau}{U}\left(11.12 + 3.78\,\frac{U_w}{U_\tau} + 0.38\,\frac{U_w^2}{U_\tau^2}\right) \tag{5}$$

or

$$\frac{\theta}{\delta} = \frac{\delta^*}{\delta} - \frac{U_\tau^2}{U^2}\left(11.12 + 3.78\,\frac{U_w}{U_\tau} + 0.38\,\frac{U_w^2}{U_\tau^2}\right). \tag{6}$$

Furthermore, if equation 2 is written as

$$\frac{U_\tau}{U} = \frac{\delta^*}{\delta}\left(\frac{1}{2.30 + 0.50\,\frac{U_w}{U_\tau}}\right)$$

and this is substituted for $\frac{U_\tau}{U}$ in equation 6, then

$$\frac{\theta}{\delta} = \frac{\delta^*}{\delta} - \frac{\delta^{*2}}{\delta^2}\,\frac{\left(11.12 + 3.78\,\frac{U_w}{U_\tau} + 0.38\,\frac{U_w^2}{U_\tau^2}\right)}{\left(2.30 + 0.5\,\frac{U_w}{U_\tau}\right)^2}\ ;$$

or, letting

$$f_1 = \frac{\left(11.12 + 3.78\,\frac{U_w}{U_\tau} + 0.38\,\frac{U_w^2}{U_\tau^2}\right)}{\left(2.30 + 0.5\,\frac{U_w}{U_\tau}\right)^2}\ , \tag{7}$$

$$\frac{\theta}{\delta} = \frac{\delta^*}{\delta} - f_1\,\frac{\delta^{*2}}{\delta^2}\ . \tag{8}$$

In a similar manner, it can be shown that

$$\frac{\delta^{**}}{\delta} = 2\,\frac{\delta^*}{\delta} - 3f_1\,\frac{\delta^{*2}}{\delta^2} + f_2\,\frac{\delta^{*3}}{\delta^3} \tag{9}$$

where

$$f_2 = \frac{\left(71.40 + 18.80 \frac{U_w}{U_\tau} + 4.80 \frac{U_w^2}{U_\tau^2} + 0.30 \frac{U_w^3}{U_\tau^3}\right)}{\left(2.30 + 0.50 \frac{U_w}{U_\tau}\right)^3} .$$

The term $\frac{U_w}{U_\tau}$ has been shown (Reference 3) to range from zero in turbulent pipe flow and to approach infinity at separation. Therefore, the functions f_1 and f_2 have been evaluated over that range and are presented in Figure 2. It is significant that there is little variation of either f_1 or f_2 with changes in $\frac{U_w}{U_\tau}$, particularly after $\frac{U_w}{U_\tau}$ becomes moderately large, for example, of the order of 10.0.

With this knowledge of f_1 and f_2, one can determine the variations of $\frac{\theta}{\delta}$ and $\frac{\delta^{**}}{\delta}$ with $\frac{\delta^*}{\delta}$ by the use of equations 8 and 9, respectively. These variations are presented in Figures 3 and 4.

The curves shown in these figures are drawn with the assumption that any value of $\frac{\delta^*}{\delta}$ is possible at a given value of $\frac{U_w}{U_\tau}$. This assumption, however, is not at all valid, and therefore limits must be established which delineate the allowable values of $\frac{\delta^*}{\delta}$ for each value of $\frac{U_w}{U_\tau}$.

As an example of the limitations, consider the separation profile where $\frac{U_w}{U_\tau} \rightarrow \infty$. In this case $U_\tau = 0$ and $U_w = U$. Inserting these values into the modified form of equation 2, one obtains

$$\frac{\delta^*}{\delta} = \frac{U_\tau}{U}\left(2.30 + 0.5 \frac{U_w}{U_\tau}\right) \qquad (11)$$

$$= 2.30 \frac{U_\tau}{U} + 0.5 \frac{U_w}{U}$$

or

$$\frac{\delta^*}{\delta} = 0.5 \quad \left(\text{FOR } \frac{U_\tau}{U} = 0 \text{ and } \frac{U_w}{U} = 1\right).$$

This value of $\frac{\delta^*}{\delta}$ is the only value possible for the case of $\frac{U_w}{U_\tau} \rightarrow \infty$ since the only value $\frac{U_\tau}{U}$ can take in this case is $\frac{U_\tau}{U} = 0$. Therefore, no points of the curves shown in Figures 3 and 4 for $\frac{U_w}{U_\tau} \rightarrow \infty$ are allowable, except that one point where $\frac{\delta^*}{\delta} = 0.50$. In all other cases, $\frac{U_\tau}{U}$ can vary between some limits yet to be defined. It is now necessary to delineate the limits in order that the allowable variations in $\frac{\delta^*}{\delta}$ for a given value of $\frac{U_w}{U_\tau}$ may be determined by use of equation 11.

In this regard, consider first the maximum attainable value of $\frac{U_\tau}{U}$. This value has been determined in Reference 3 as a function of $\frac{U_w}{U_\tau}$ from a consideration of the properties of the laminar sublayer. The maximum value of $\frac{U_\tau}{U}$ so determined is given as

$$\left(\frac{U_\tau}{U}\right)_{max} = \frac{1}{17.20 + \frac{U_w}{U_\tau}} . \tag{12}$$

With this relation and equation 11, the maximum possible values of $\frac{\delta^*}{\delta}$ can be determined as a function of $\frac{U_w}{U_\tau}$:

$$\left(\frac{\delta^*}{\delta}\right)_{max} = \frac{2.30 + 0.5\frac{U_w}{U_\tau}}{17.20 + \frac{U_w}{U_\tau}} . \tag{13}$$

The limit of this relation as $\frac{U_w}{U_\tau} \to \infty$ is seen to be

$$\left(\frac{\delta^*}{\delta}\right)_{max} = 0.5 \quad \text{when} \quad \frac{U_w}{U_\tau} \to \infty$$

as previously determined.

In this manner, the upper limit of each of the curves in Figures 3 and 4 can be determined. The curves of Figures 3 and 4 have been redrawn within these limits and are shown in Figures 5 and 6.

It remains now to determine the minimum allowable values of $\frac{\delta^*}{\delta}$, and any relationship between θ and $\frac{U_\tau}{U}$ can be used for this purpose. Using the relationship given in Reference 3,

$$\log R_\theta = \frac{1}{5.6}\left[\frac{U}{U_\tau} - \frac{U_w}{U_\tau}\right] - \log\left[\frac{10}{\left(2.30 + 0.5\frac{U_w}{U_\tau}\right) - \frac{U_\tau}{U}\left(11.12 + 3.78\frac{U_w}{U_\tau} + 0.38\frac{U_w^2}{U_\tau^2}\right)}\right], \tag{14}$$

the values of $\frac{U_\tau}{U}$ can be determined for any values of R_θ and $\frac{U_w}{U_\tau}$. For the value of $\frac{U_\tau}{U}$, thus determined to be a minimum, the chosen value of R_θ must be a maximum for the given value of $\frac{U_w}{U_\tau}$. At present there is no reason to suspect that a physically dictated upper limit exists for R_θ ; however, it is not unreasonable to expect a practical upper limit to the values of R_θ encountered in practice. An extremely thick boundary layer, δ greater than ten feet, measured at the aft end of an airship in flight (Reference 5), was seen to have a value of $R_\theta < 10^6$; therefore the value $R_\theta = 10^6$ will be arbitrarily chosen as the highest value normally expected in the vast majority of measurements.

With $R_{\theta_{max}}$ thus defined, the minimum values of $\frac{U_\tau}{U}$ can be computed for any given $\frac{U_w}{U_\tau}$. These lower limit values of $\frac{U_\tau}{U}$ have then been inserted into equation 11, and the lower limits of $\frac{\delta^*}{\delta}$ have been determined. Again, the curves of Figures 5 and 6 have been redrawn within these lower limits, and the new curves are presented in Figures 7 and 8.

Examination of these figures reveals that the relations between $\frac{\delta^*}{\delta}$ and $\frac{\theta}{\delta}$ or $\frac{\delta^{**}}{\delta}$ are very nearly independent of $\frac{U_w}{U_\tau}$. This relative independence of $\frac{U_w}{U_\tau}$ is further seen in Figure 9. Here equations 8 and 9 were altered to yield

$$H = \frac{1}{\left(1 - f_1 \frac{\delta^*}{\delta}\right)} \tag{15}$$

and

$$\bar{H} = \frac{\left(2\frac{\delta^*}{\delta} - 3f\frac{\delta^{*2}}{\delta^2} + f_2\frac{\delta^{*3}}{\delta^3}\right)}{\left(1 - f_1\frac{\delta^*}{\delta}\right)} \tag{16}$$

and are shown as a function of $\frac{U_w}{U_\tau}$ within the same limits as Figure 7 and 8. It should be noticed that the maximum value of H predicted by equation 15 is about 4.13. This value corresponds to the case of $\frac{U_w}{U_\tau} \rightarrow \infty$ or $\frac{\delta^*}{\delta} = 0.5$. In Figure 10, a mean line drawn through the curves of Figure 9 is compared with other existing relationships between H and $\bar{H}$.

COMPARISON WITH EXPERIMENT AND CONCLUDING REMARKS

As a test of the validity of the previously derived relations, the curves of Figures 7 and 8 are compared with the experiment in Figures 11 and 12. In this case the large number of segments have been replaced by a single mean line for simplicity. The data used for this comparison ranged from pipe profiles, measured in small pipes, to very thick profiles, measured near the separation point of the flow over an airstrip. The good agreement between these various experiments and the present theory indicates a nearly single-valued dependence of $\frac{\theta}{\delta}$ and $\frac{\delta^{**}}{\delta}$, and therefore H and $\bar{H}$ upon $\frac{\delta^{*}}{\delta}$.

It should be mentioned that the relatively high value of H at separation, which is predicted by the present theory, results from the assumption that the mean flow at separation takes the shape of the wake profile given by Coles. In an actual flow, however, turbulent fluctuations may cause instantaneous flow separation at values of H considerably lower than predicted here. An experimental search for the highest attainable value of H would be of interest.

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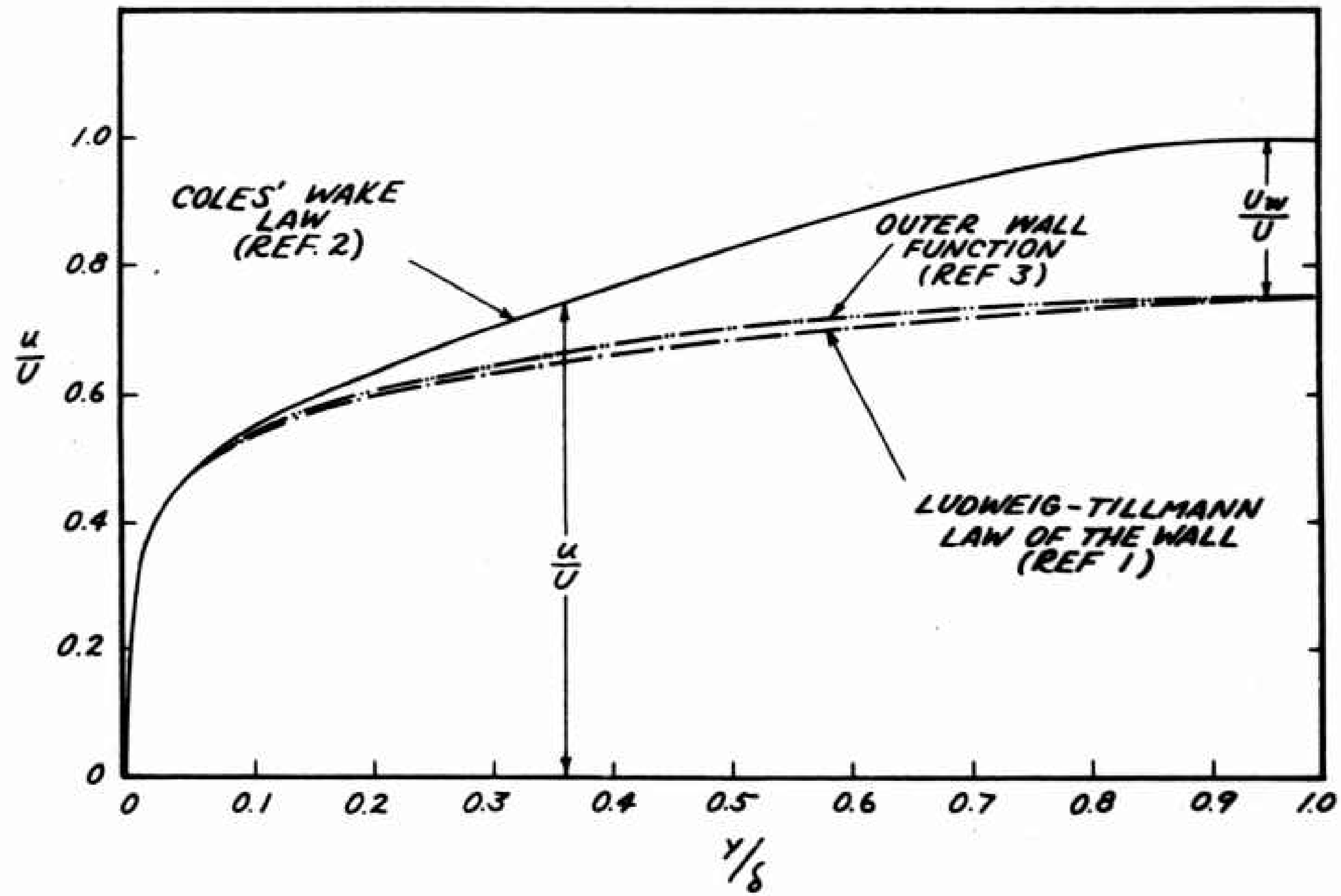

Figure 1. Present Nomenclature for Turbulent Boundary Layer.

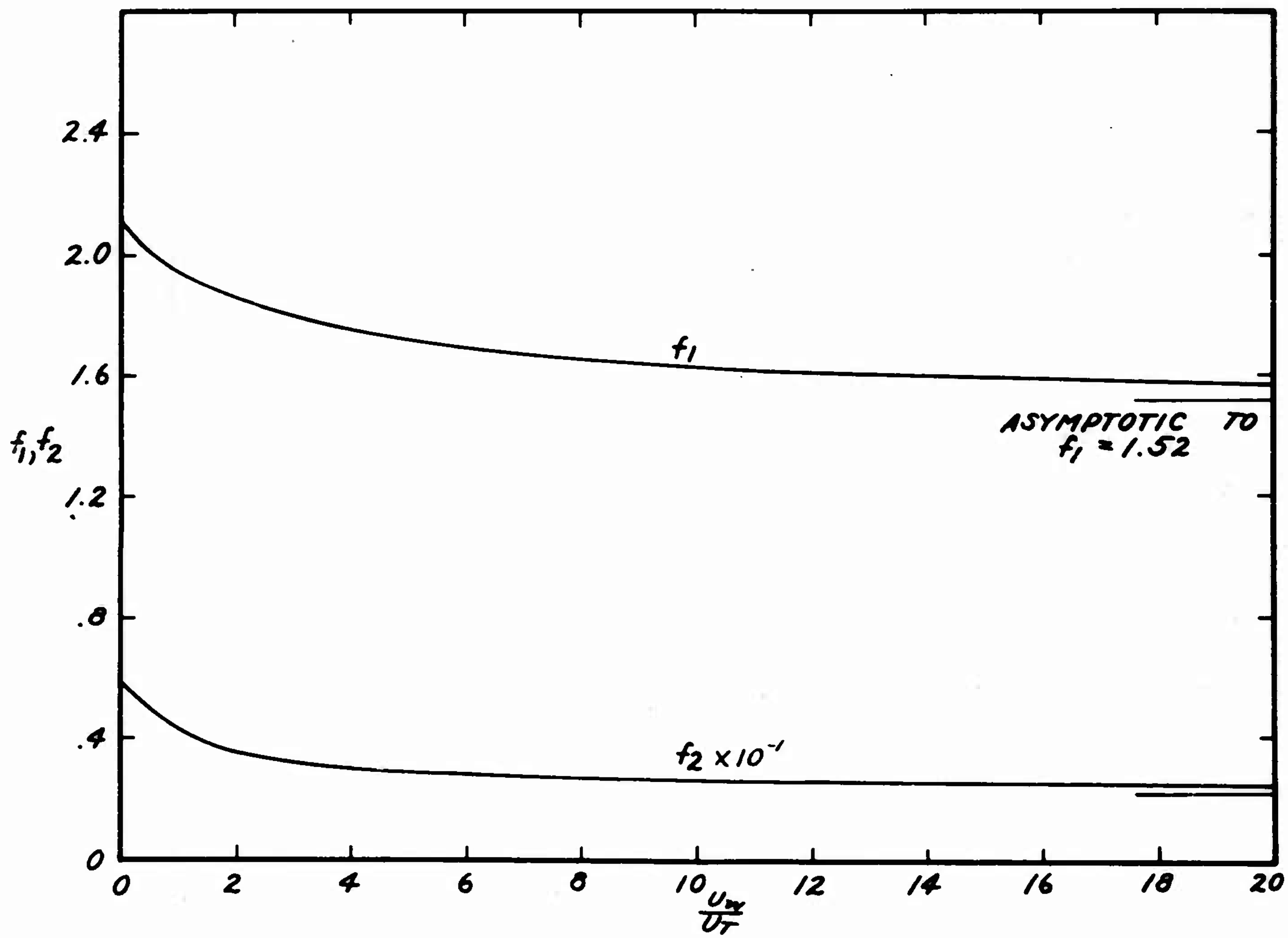

Figure 2. Evaluation of f_1 and f_2.

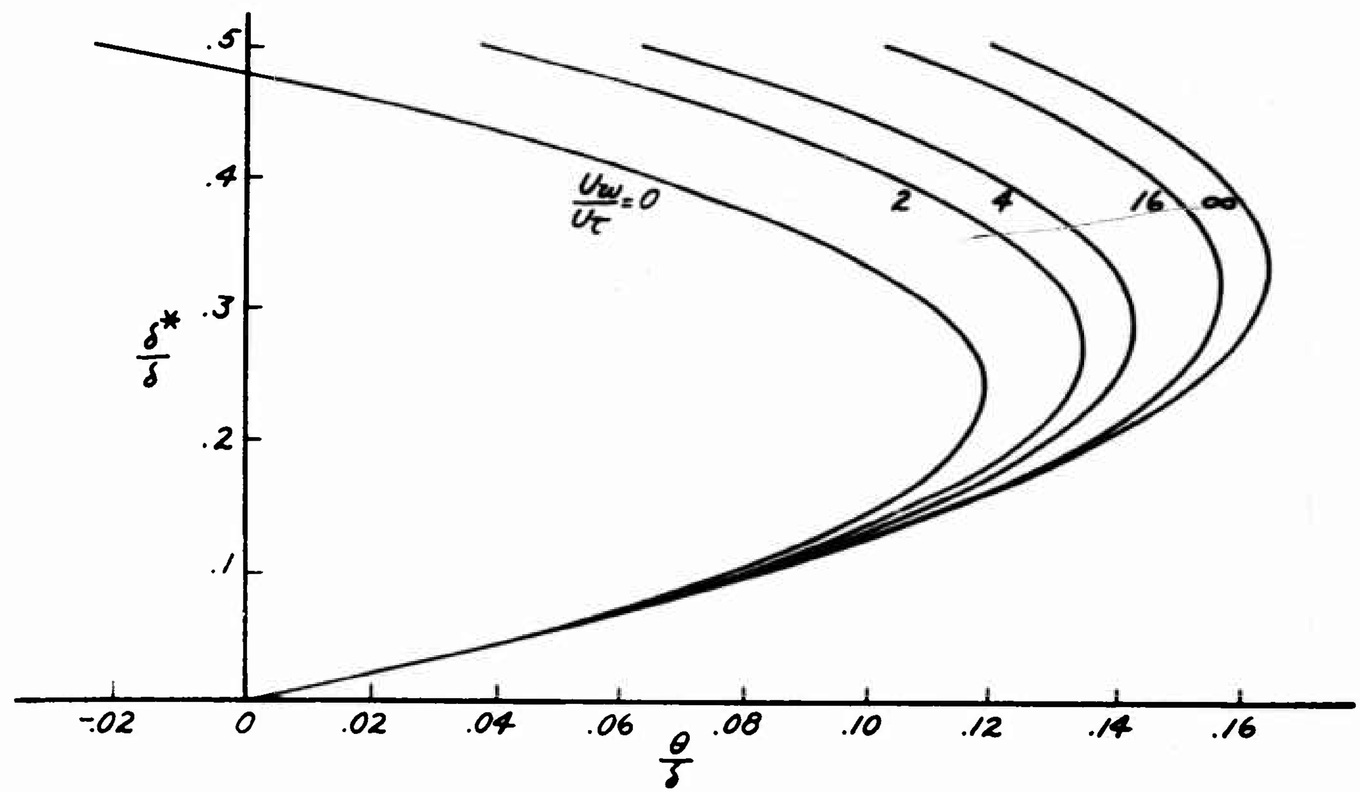

Figure 3. Theoretical Variation of $\frac{\theta}{\delta}$ with $\frac{\delta^*}{\delta}$ as a Function of $\frac{U_w}{U_\tau}$.

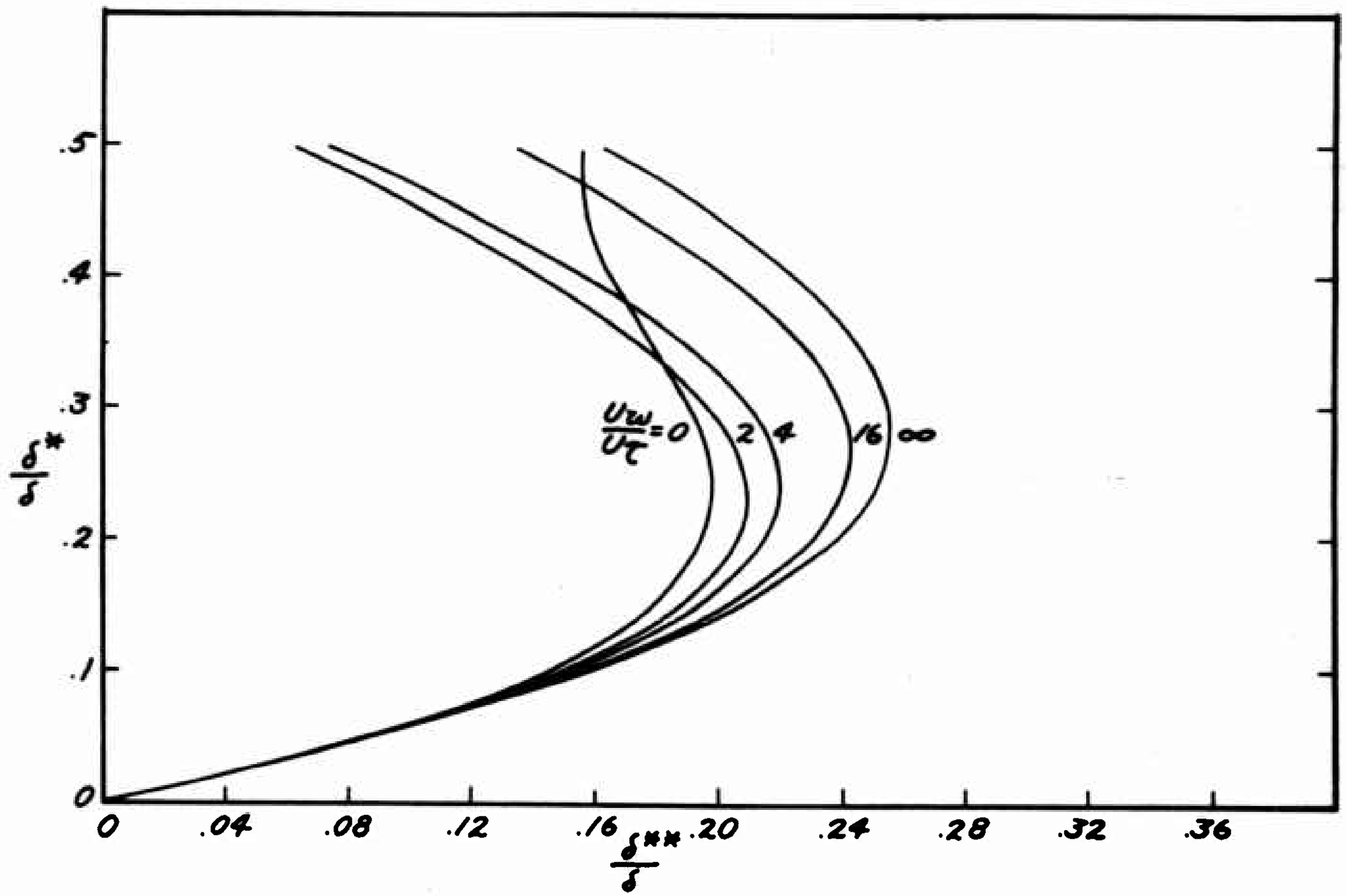

Figure 4. Theoretical Variation of $\frac{\delta^{**}}{\delta}$ with $\frac{\delta^*}{\delta}$ as a Function of $\frac{U_w}{U_\tau}$.

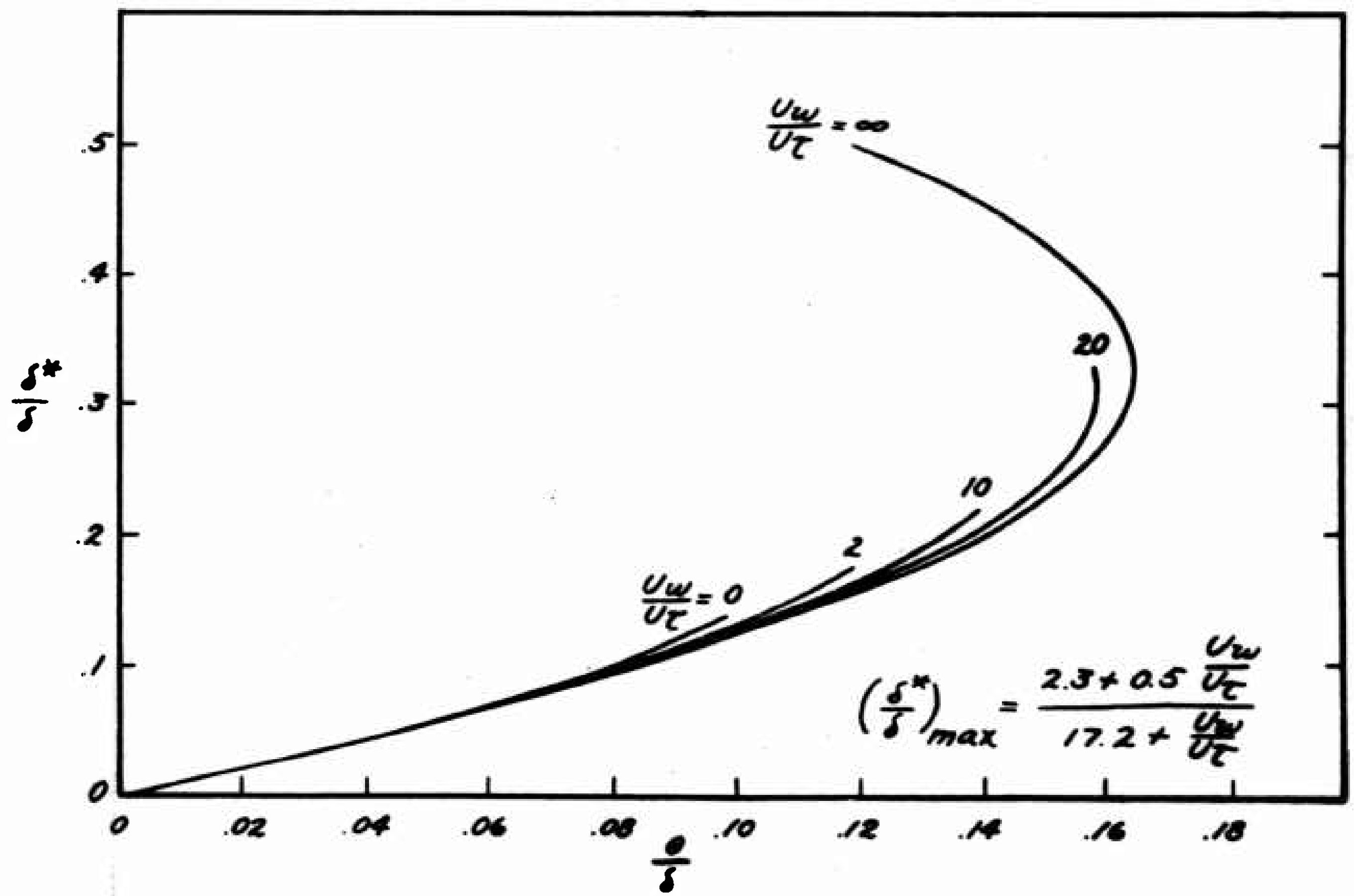

Figure 5. Upper Limits of $\frac{\delta^*}{\delta}$ as a Function of $\frac{U_w}{U_\tau}$.

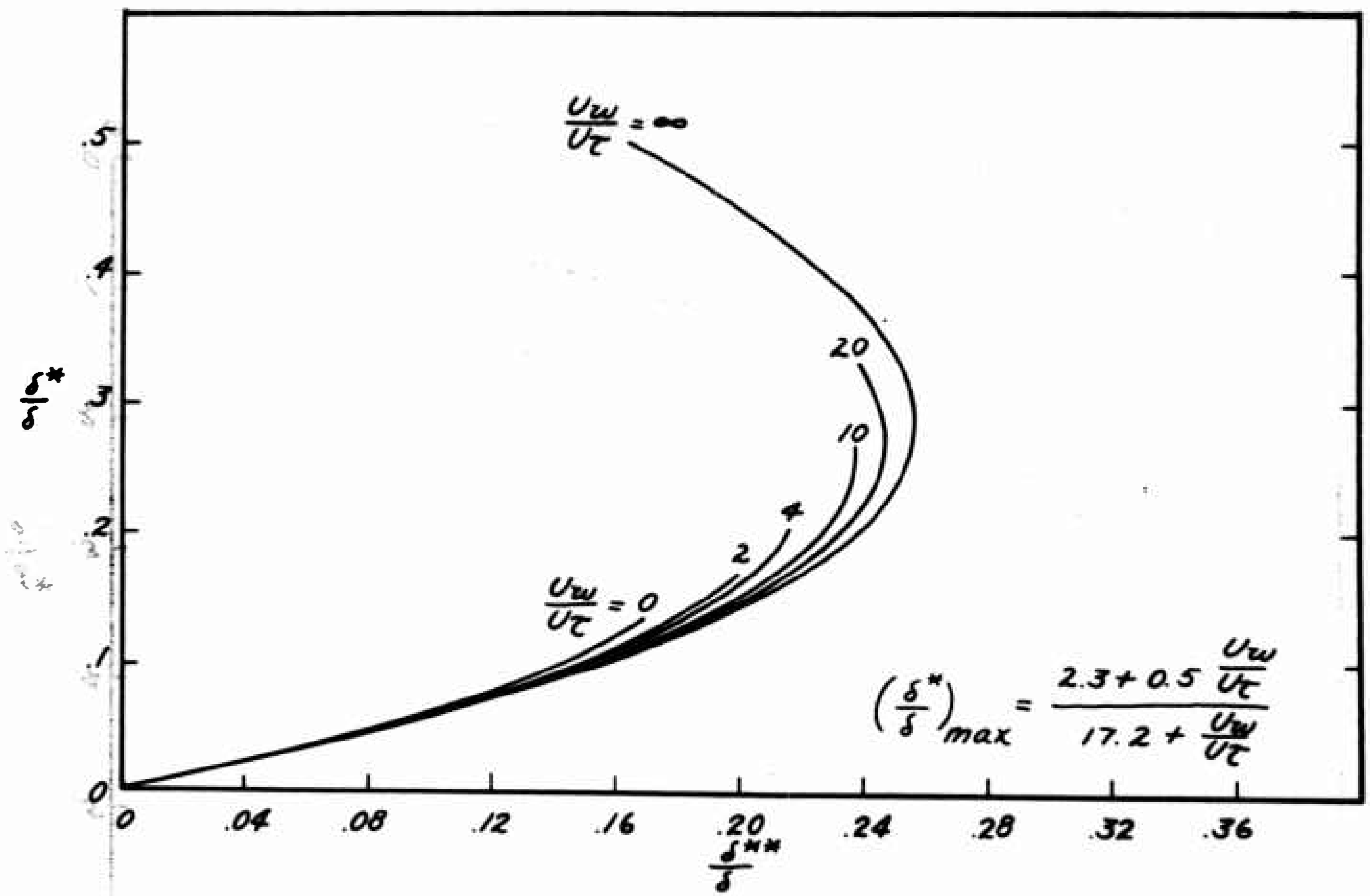

Figure 6. Upper Limits of $\frac{\delta^*}{\delta}$ as a Function of $\frac{U_w}{U_\tau}$.

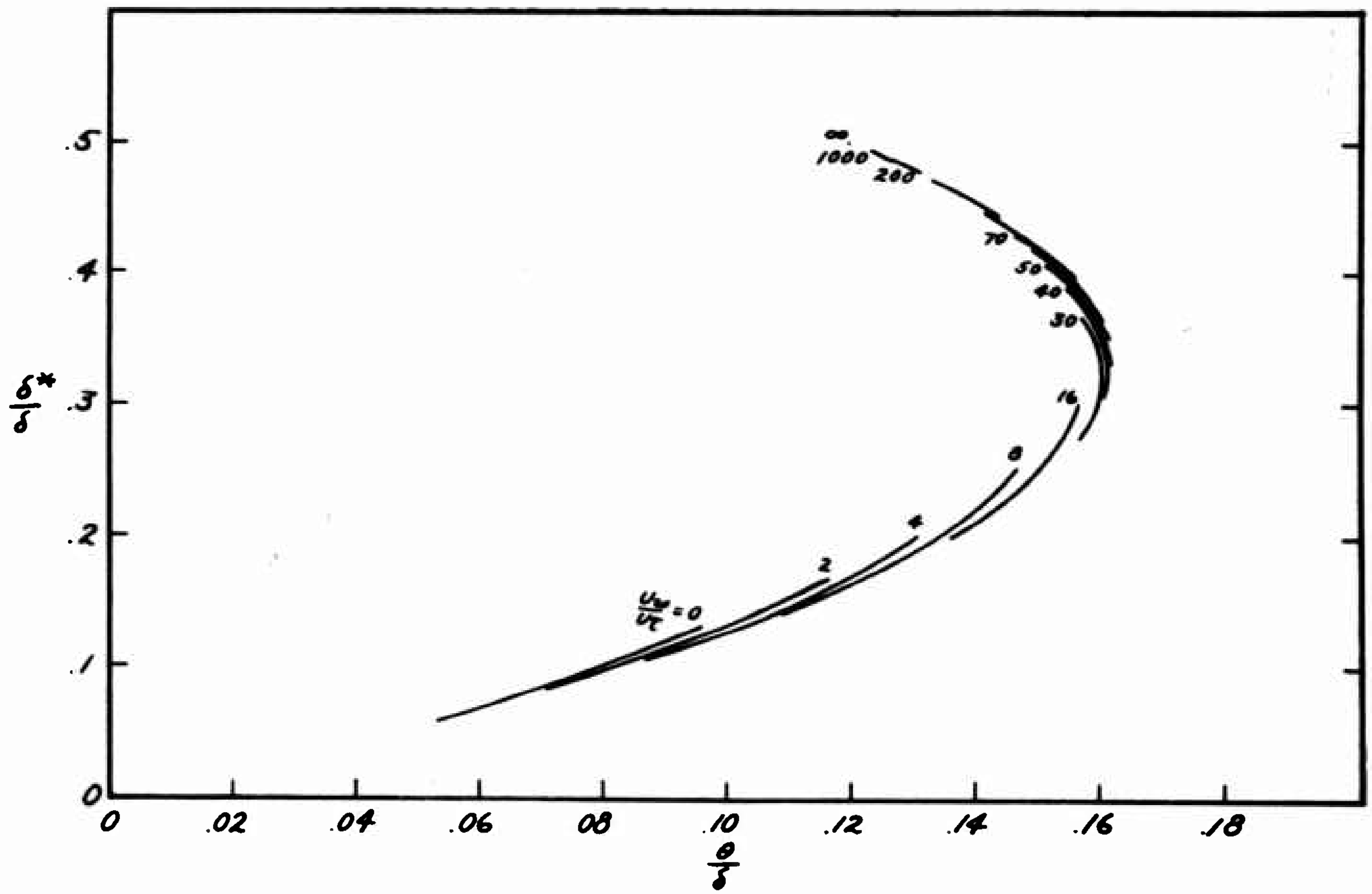

Figure 7. Relation Between $\frac{\delta^*}{\delta}$ and $\frac{\theta}{\delta}$.

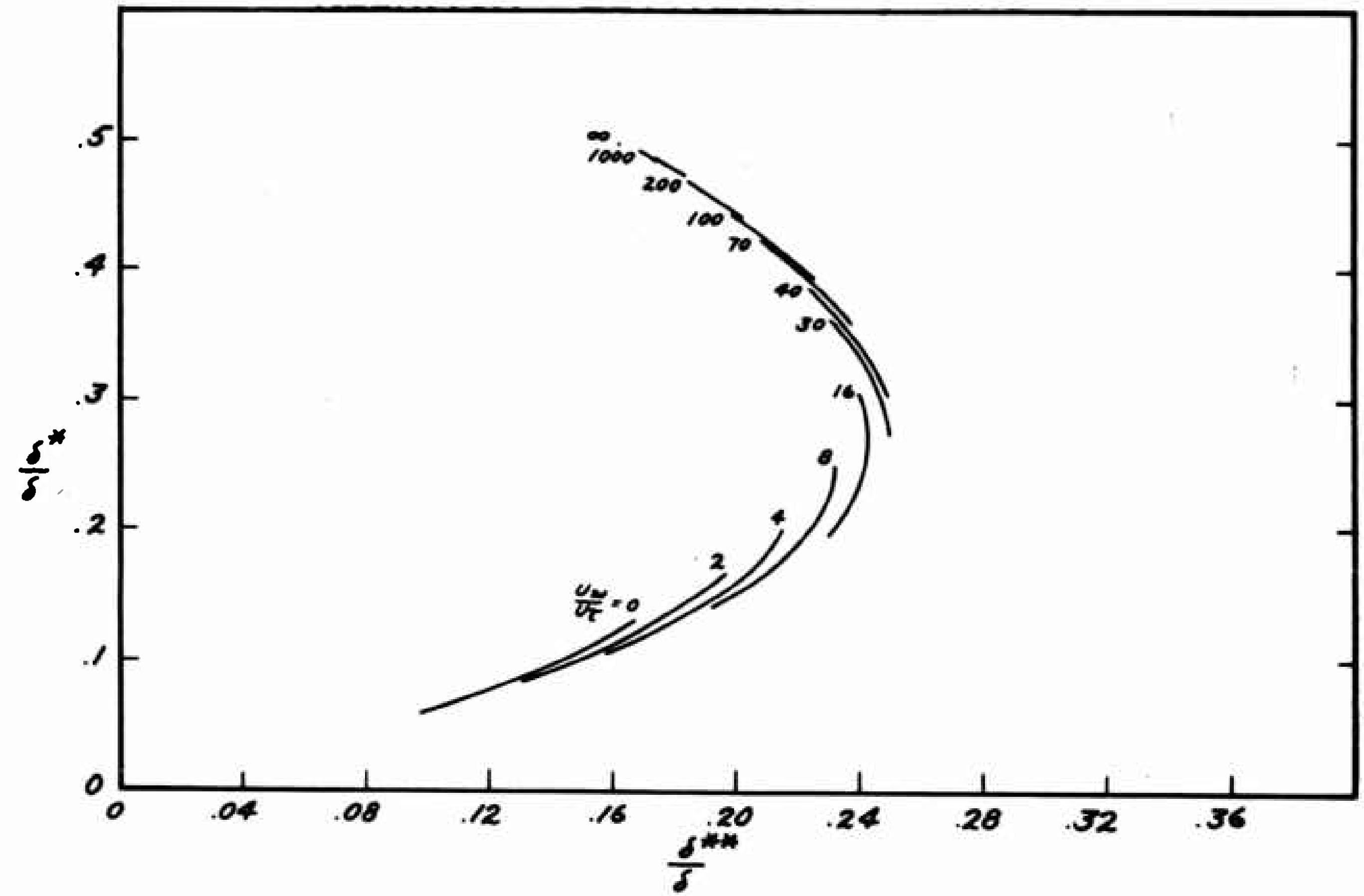

Figure 8. Relation Between $\frac{\delta^*}{\delta}$ and $\frac{\delta^{**}}{\delta}$.

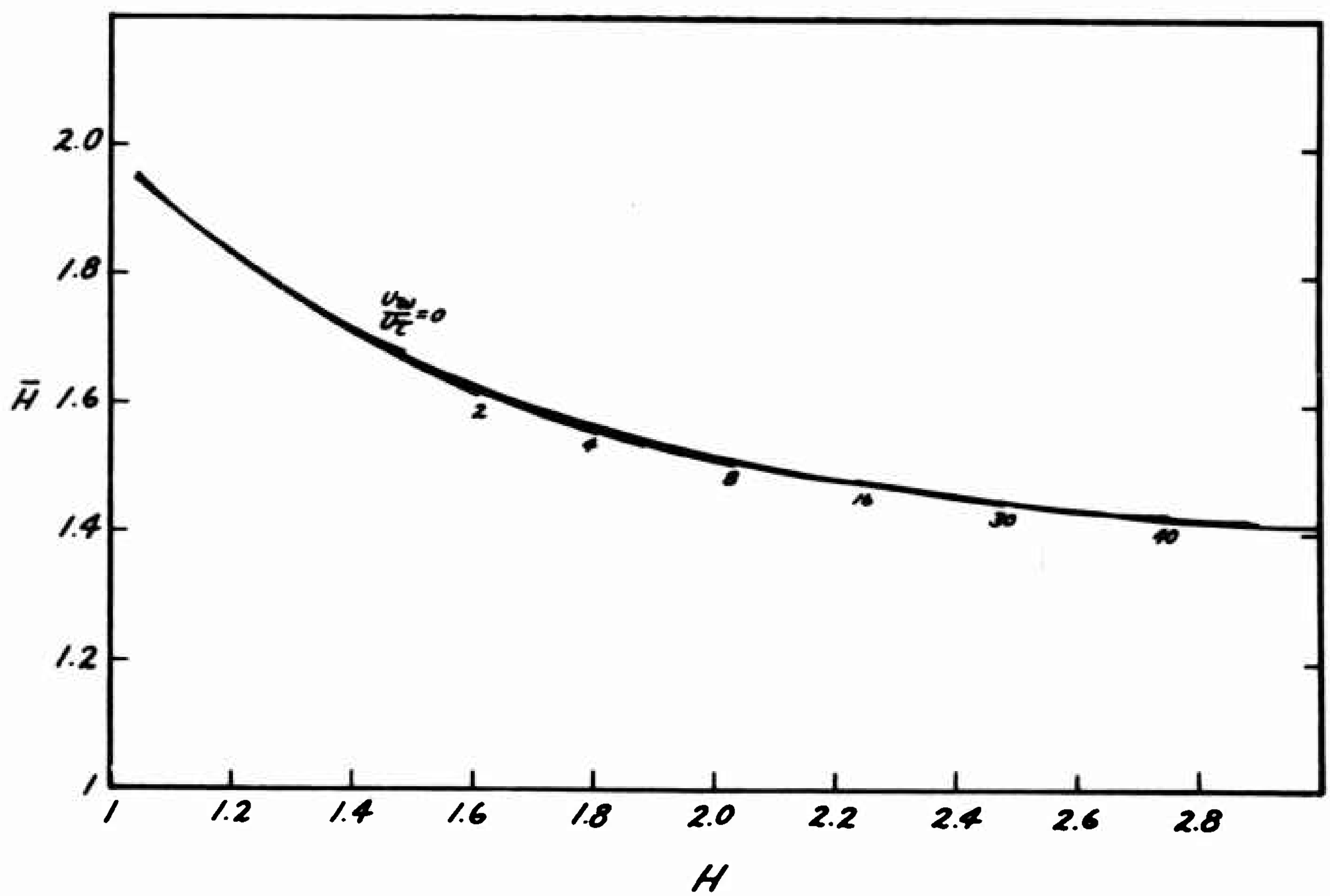

Figure 9. Relation Between H and $\bar{H}$ as a Function of $\frac{U_\omega}{U_\tau}$.

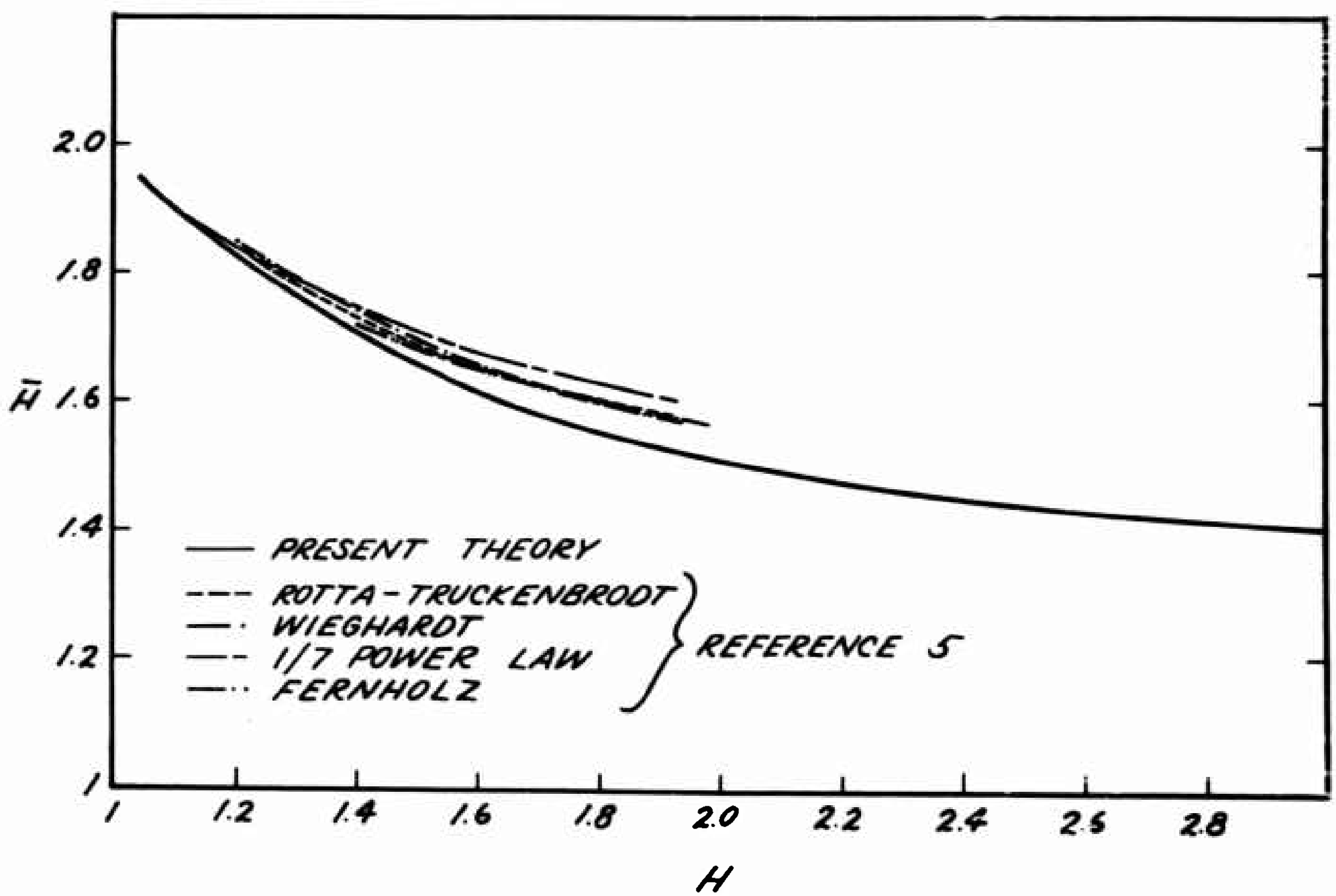

Figure 10. Comparison of Relation Between H and $\bar{H}$.

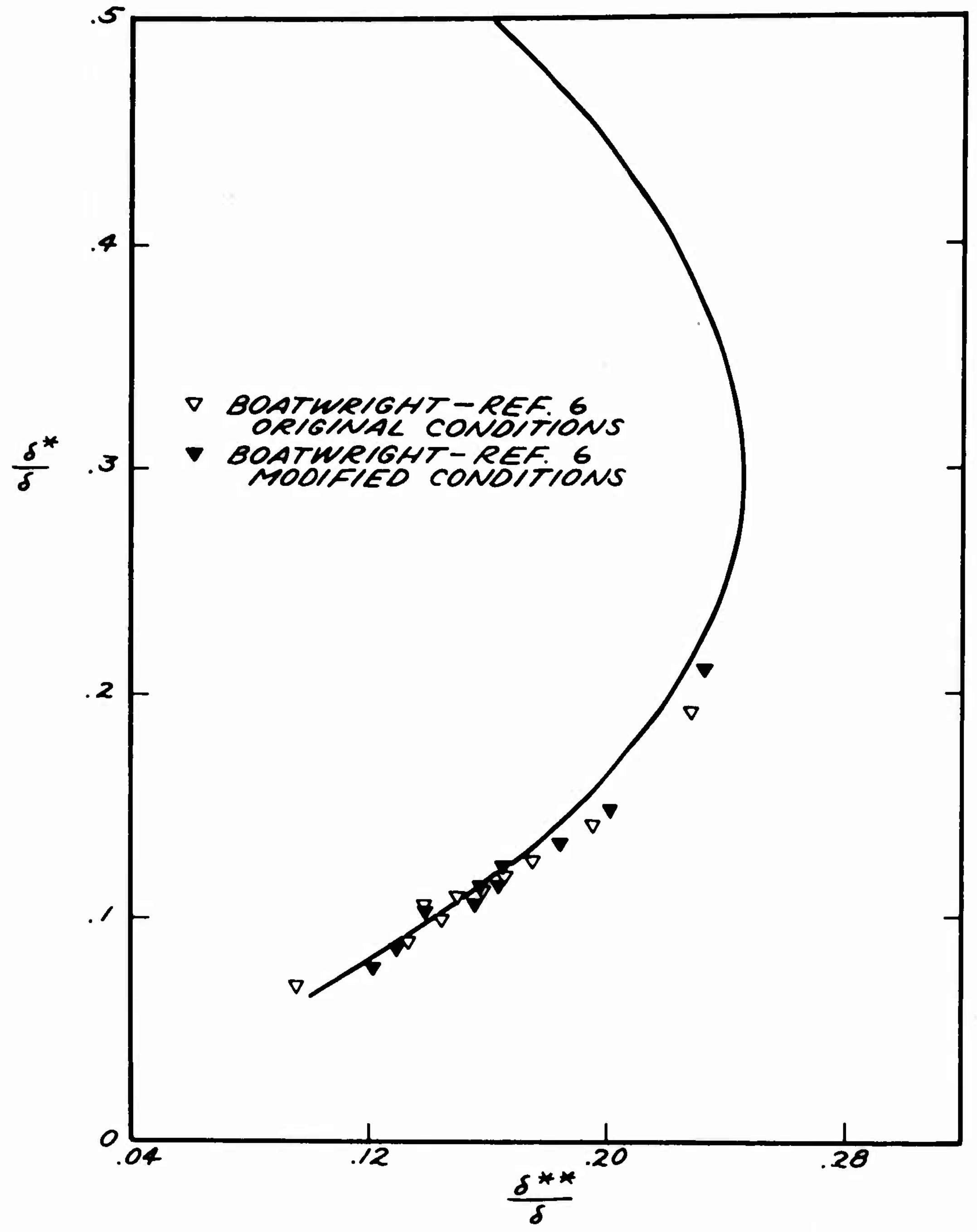

Figure 11. Comparison of Theory with Experiment.

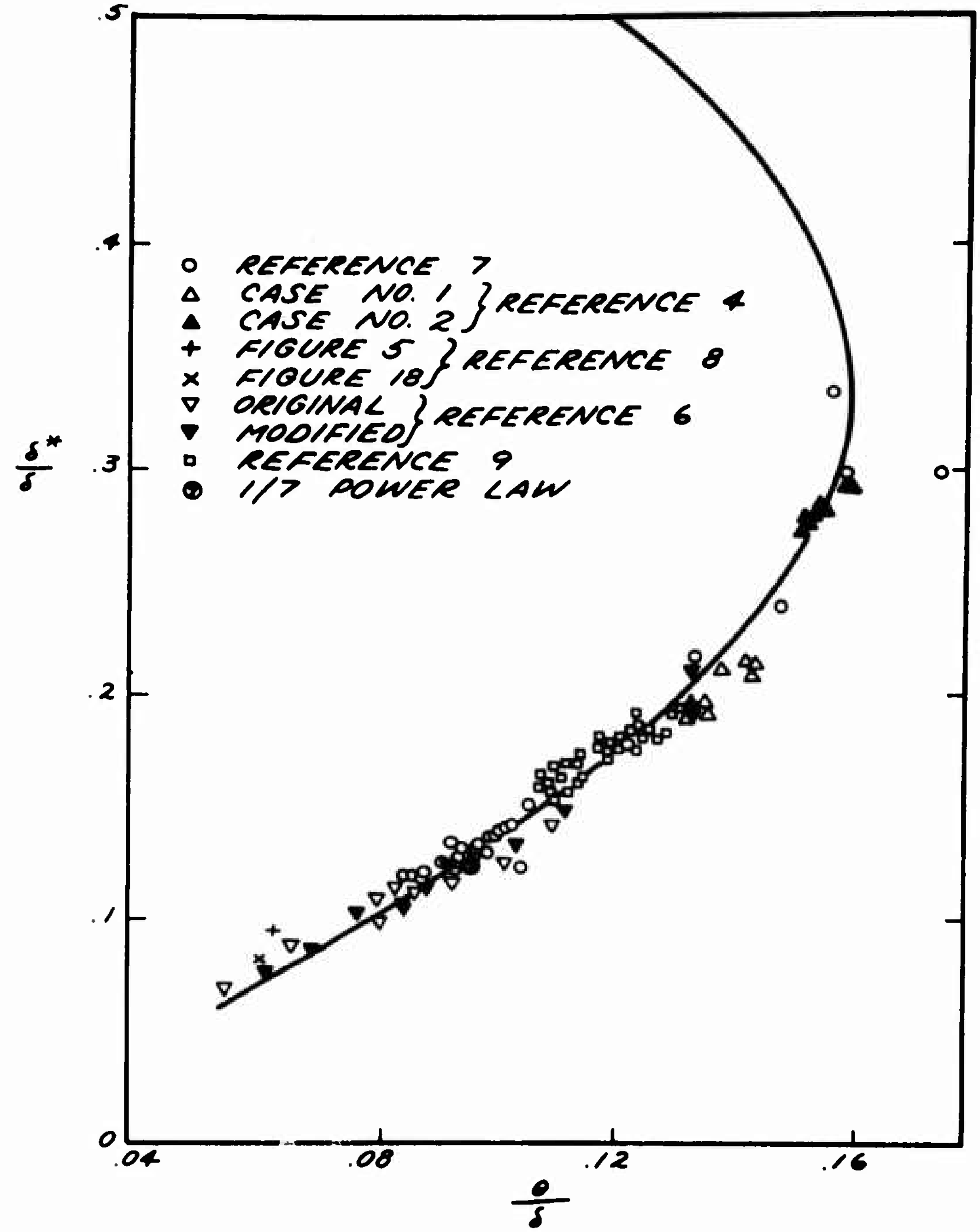

Figure 12 Comparison of Theory with Experiment.

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1. Turbulent Boundary Layer

Unclassified Report

From an examination of the turbulent boundary layer profile, several relations are derived relating

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$$\frac{\delta^{**}}{\delta} = 2\frac{\delta^*}{\delta} - 3f_1 \frac{\delta^{*2}}{\delta^2} + f_2 \frac{\delta^{*3}}{\delta^3}$$

$$H = \frac{1}{1 - f_1 \frac{\delta^*}{\delta}}$$

$$\bar{H} = \frac{2\frac{\delta^*}{\delta} - 3f_1 \frac{\delta^{*2}}{\delta^2} + f_2 \frac{\delta^{*3}}{\delta^3}}{\frac{\delta^*}{\delta} - f_1 \frac{\delta^{*2}}{\delta^2}}$$

These relations are shown to have limits of application, and these limits are defined. Finally, the theories are compared with experimental data and show good agreement.

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